1) Release mechanism.
2) Jack connections for test leads.
3) Release button.
4) Contact plates.
5) Depression for placement of ball.
6) Magnetic holder for extra ball.
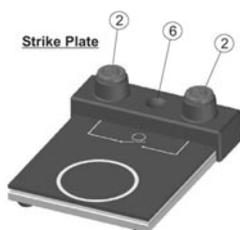
Description

This apparatus is designed to be used with an Electronic Counter to determine the fall time for a freely falling steel ball. The acceleration of gravity g can be computed when the path length s and the fall time t are known: $s = 1/2 \cdot g \cdot t^2$.

The apparatus consists of a release mechanism which also acts as the start switch, a strike plate which acts as the stop switch and gold-plated steel balls. Two ea. 12 mm and two ea. 16 mm diameter balls are supplied as well as a tennis ball adjusted to have the same mass as a 12 mm diameter steel ball.

The release mechanism has two contact plates (4) which are shorted when the steel ball to be measured is in place and ready to drop. The steel ball is held in place by means of a spring loaded neodymium magnet. The steel balls have been gold plated to ensure good electrical contact. They should be kept clean using alcohol or petrolum ether.

The neodymium magnet can be removed rapidly from the ball by means of the spring loaded release (1). This will cause the steel ball to drop away from the contact plates thus opening the counter start circuit just as the fall begins. When the ball hits the strike plate a second switch is closed thus stopping the counting process.

The release mechanism is provided with two jack connectors for connecting safety jack leads to the Counter start switch input. A 10 mm diameter mounting rod for supporting the mechanism on a lab stand is supplied. The strike plate is also provided with two jack connectors for attaching this unit to the Counter stop input.
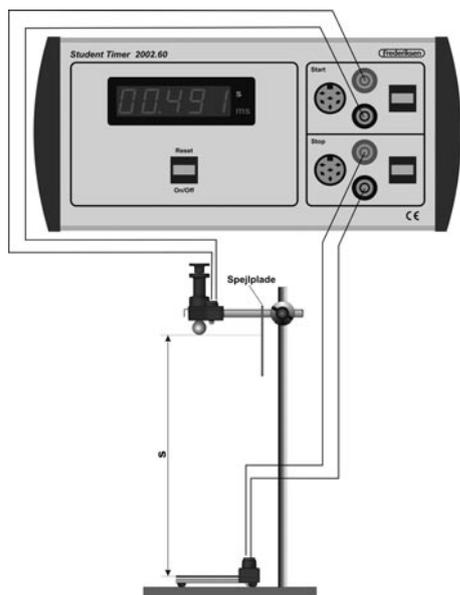
Frederiksen®

Figure 2.

Operation

The apparatus is assembled as shown in Figure 2. The release mechanism is readied by compressing the spring release (1). The steel ball is placed in the depression (5) between the contact plates (4) on the release mechanism. The steel ball is released by pressing the push button. The connection between the plates is broken, and the counter starts. When the steel ball hits the strike plate placed vertically below the release point the counter is stopped.

The length of the fall s can be measured using a ruler. Measure the distance from the bottom of the ball when it is mounted in the release mechanism to the strike plate. The mirror provided can be helpful when making this measurement to avoid parallax error. See Figure 2.

Important!

In order to ensure as good a connection as possible both the contact plates and the steel ball are gold plated and must be kept absolutely clean. They can be cleaned using an organic solvent. Some people have moist fingers, and this can make it difficult to achieve good contact. The use of thin cotton gloves can be helpful in this case.

Maintenance

The release mechanism spring should be relaxed during storage. It is important to keep the ball and the contact plates clean.

Essential accessories:

1 ea. Student timer 2002.60

4 ea. Test leads

1 ea. Retort stand 0001.00

1 ea. Retort stand rod 0008.00

1 ea. Bosshead 0023.00

Spare parts

1980.11   Extra set of gold plated steel balls.

Table for copying:

Fall path length, s =_____.   Mass of ball, m = _____

Fall time (ms)	g=(2s)/t

Calculate the average value of g: _____.

Compute the deviation from the table value for g:

FREELY FALLING BODIES

Lab exercise

Purpose

The goal of the exercise is to measure falling times for freely falling bodies in order to determine the acceleration of gravity. The experiment will also involve bodies where air resistance is significant. The Frederiksen free fall apparatus (1980.10) is used to illuminate the agreement between theory and experiment.

Equipment

1 ea. Free fall apparatus including steel balls and ping pong ball. 1980.10
1 ea. Student timer 2002.60
4 ea. Test leads
1 ea. Retort stand 0001.00
1 ea. Retort stand rod 0008.00
1 ea. Bosshead 0023.00

Setup
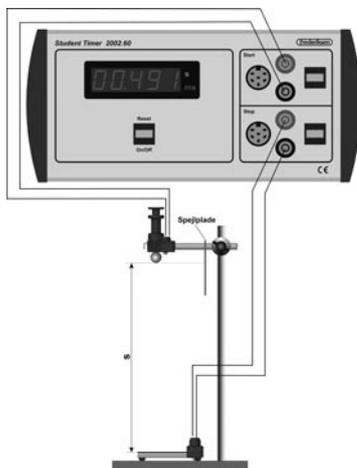
Theory
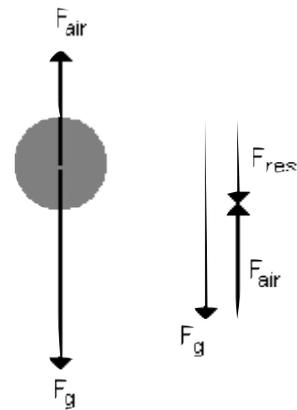
The above figure shows a body falling under the action of the earth's gravity. The acceleration of gravity $g = 9{,}82 \ m/s^2$. When the speed of the falling body increases the opposing force due to air resistance increases. It is assumed here that air resistance is proportional to the square of the speed: $F = k \cdot v^2$.

The resultant force F_{RES} on the body is given by:

$$F_{RES} = m \cdot g - k \cdot v^2$$

Where it is assumed that the positive direction is downward.

Without air resistance

We assume first that $k = 0$, i.e. That the force of air resistance is insignificant. Thus

$$F_{RES} = m \cdot g = m \cdot dv/dt$$

Where we have used Newton's second law: $F = m \cdot a = m \ dv/dt$. Thus:

$$dv/dt = g$$

Which has the complete solution

$$v(t) = g \cdot t + c$$

Where c is an arbitrary constant. If the motion begins at $t = 0$, then this constant corresponds to the initial speed v_0:

$$v(t) = g \cdot t + v_0$$

Note that $v = g \cdot t$ when the initial speed $v_0 = 0$. Because the speed $v = ds/dt$, then the general formula is

$$ds/dt = g \cdot t + v_0$$

Which has the solution

$$s(t) = \tfrac{1}{2} \cdot g \cdot t^2 + v_0 \cdot t + s_0$$

Where the constant of integration s_0 in this case corresponds to the initial position.

If one assumes that the body is at rest ($v_0 = 0$) at the initial position ($s_0 = 0$) at time $t = 0$, the following result:

$$s = \tfrac{1}{2} \cdot g \cdot t^2 \iff t = \sqrt{\frac{2 \cdot s}{g}}$$

Exercise

Measure corresponding values of the fall distance s and the time for the fall t. Check by graphing s as a function of t^2 whether this model is a good description of the motion. The slope of the graph should correspond to $\tfrac{1}{2} \cdot g$ where g is the acceleration of gravity.

s/m	t/s

Free fall with air resistance

For bodies having a low density (e.g. A ping pong ball) air resistance becomes significant for the motion. I.e. It is no longer possible to set k = 0. In this case the force equation becomes:

$$F_{RES} = m \cdot g - k \cdot v^2 = m \cdot dv/dt$$

Where we again use Newton's second law. Note that the constant k in the term describing air resistance is dependent upon the cross sectional area A of the falling body, the density D of the medium (air) as well the form and surface of the object characterized by C_W. The constant C_W:

$$k = \tfrac{1}{2} \cdot D \cdot A \cdot C_W.$$

The force equation is an equation in two variables: the speed v and time t. They can be separated:

$$\frac{m\,dv}{mg - kv^2} = dt$$

The equation can be solved by integrating both sides, integrating the speed from the initial value $v_0 = 0$ and the time from t = 0 to the time t.

$$\int_0^v \frac{m\,dv}{mg - kv^2} = \int_0^t dt$$

By integration we obtain:

$$\frac{m}{2\sqrt{mgk}} \cdot \ln\left[\frac{\sqrt{mg} + \sqrt{k} \cdot v}{\sqrt{mg} - \sqrt{k} \cdot v}\right] = t$$

Multiplying and reducing:

$$e^{2\sqrt{gk/m} \cdot t} = \left[\frac{\sqrt{mg} + \sqrt{k} \cdot v}{\sqrt{mg} - \sqrt{k} \cdot v}\right]$$

Take the exponential function exp on both sides of the equation:

$$e^{2\sqrt{gk/m} \cdot t} = \left[\frac{\sqrt{mg} + \sqrt{k} \cdot v}{\sqrt{mg} - \sqrt{k} \cdot v}\right]$$

This equation can now be solved for the speed v as a function of the time t:

Which is the result sought for v(t). Using a spreadsheet s(t) can be found by using the difference equation $s_1 = s_0 + v(t_1) \cdot \Delta t$. A time interval Δt og about 0.05 s will usually be appropriate when analyzing data from the free fall apparatus.

Transition to low air resistance

It can now be shown that for small values of the coefficient k this formula is identical to the equation $v(t) = g \cdot t$ for motion without air resistance. This can be done by exchanging the exponential function with the two first terms of the Taylor series ($e^X \approx (1 + X)$). For small values of k the exponent is close to zero so that so that the speed equation can be rewritten:

$$v \approx \sqrt{\frac{mg}{k}} \cdot \left[\frac{1 + 2\sqrt{gk/m} \cdot t - 1}{1 + 2\sqrt{gk/m} \cdot t + 1}\right] = \sqrt{\frac{mg}{k}} \cdot \left[\frac{2\sqrt{gk/m} \cdot t}{2\sqrt{gk/m} \cdot t + 2}\right]$$

Where we have made use of the fact that the denominator approaches unity for small values of k. In this manner it is apparent that the exact expression reduces to $v = g \cdot t$ for $k \cdot 0$.

Exercise

Prepare a spreadsheet with the exact expression for v(t) and for s(t) using the difference equation $s_1 = s_0 + v(t_1) \cdot \Delta t$, and compare the result with the corresponding expression without air resistance. Compare (t,v) and (t,s) graphs for various values of

$$k = \tfrac{1}{2} \cdot D \cdot A \cdot C_w$$

Theory and real data can be compared using the Frederiksen free fall apparatus. The density of air D = 1,293 kg/m³ and the cross sectional area A can be determined, the value of the air resistance coefficient C_w can be found. This is readily done by checking which value of C_w in the expression for v(t) which gives the best agreement with the measured data when s(t) is computed in the spreadsheet using the difference equation.

Exercise

With knowledge of C_w it is possible to compute the terminal velocity for a falling body, for in this case

$$m \cdot g - k \cdot v^2 = 0:$$

$$v = \sqrt{\frac{m \, g}{k}}$$

This is also apparent for the limiting value of v(t) for t going towards infinity. Perform an experiment by letting the ball fall from a high spot (e.g. Top of a building). Does the object approach a constant speed.?

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